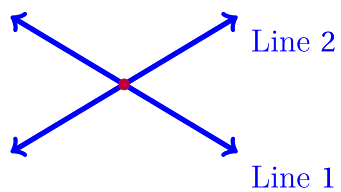
9.1-9.2: Systems of Linear Equations (Two or Three Variables)

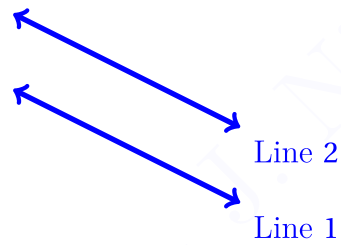
- Geometric Representation of the Solution (Two Variables)

  In case of system of two equations in two variables, each equation represents a line. The solution, if it exists, is the intersection of the two lines. Cases are:

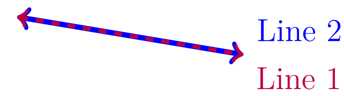
  - **An independent system** is two distinct and intersecting lines which have a **unique** point of intersection.
  - **A dependent system** is when both equations represent the same line, the solution set is **infinite** and the entire line.
  - Two parallel lines don't intersect and the system is going to be **inconsistent**. (**no solutions**)



Two lines which meet at a point.



Parallel lines don't intersect.



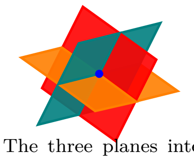
Two lines are identical.

  Now, you can complete Problems 1 - 8.

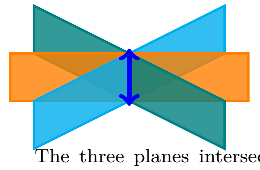
- Geometric Interpretation of Three Equations and Three Variable Systems

  In this case, every equation is a plane in three dimensional space. There a few cases:

  - The three planes meet at one point. (A unique solution)
  - The three planes meet over a line. (infinitely many solutions)
  - The three planes are the same planes, which means their intersection is a plane. (infinitely many solutions)
  - Any two or all three planes are parallel and the intersection is empty. (the system is inconsistent)
  - Each pair of planes intersect in a line but no common point between all three exists. (the system is inconsistent)

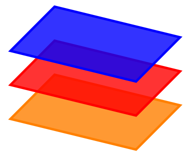


The three planes intersect in a point. (a unique solution)

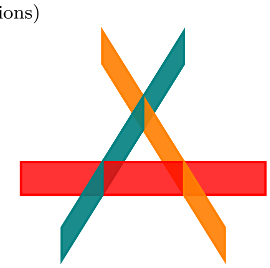


The three planes intersect in a line. (infinite solutions)

One pair of parallel planes. No common points.(inconsistent system)



Three parallel planes. No common points. (inconsistent system)



Each pair of planes intersect in a line but no points in common. (inconsistent system)

How to Solve Systems of Two or Three Variables.

- Given a system of equations, solve using the elimination method.
    - Write both equations with x, y and z variables on the left side of the equation and constants on the right.
    - Write one equation above the other, lining up corresponding variables. If any of the variables is missing in any of the equations, leave an empty spot. If one of the variables in the top equation has the opposite coefficient of the same variable in the bottom equation, add the equations together, eliminating one variable. If not, use multiplication by a nonzero number so that one of the variables in the top equation has the opposite coefficient of the same variable in the bottom equation, then add the equations to eliminate the variable.
    - Continue the above process until one equation has only one variable in it; or the elimination process results in eliminating all variables and constants ($0 = 0$); or you arrive at a contradictory equation. (Such as $0 = 1$.)
    - Solve the resulting equation for the remaining variable.
    - Substitute that value into one of the other equations and follow the above process solve for the second variable and so on.
    - Check the solution by substituting the values into the equations.
- Row Operations: The operations suggested in the process are called row operations. If any system is the result of row operations on another system, then the system is **row equivalent** to the the original. Row equivalent systems render the same solutions.

    (1) Exchange rows.

    (2) Add a multiple of row to another.

    (3) Multiply a row by a number.

1. Solve $\begin{cases} 2x & +3y & = -1 \\ 5x & -y & = 6 \end{cases}$.

2. For each of the following systems, determine if the system is inconsistent, has infinitely many solutions or has an unique solution.

   (a)
   $$\begin{cases} 2x & +3y & = -1 \\ -4x & -6y & = 6 \end{cases}$$

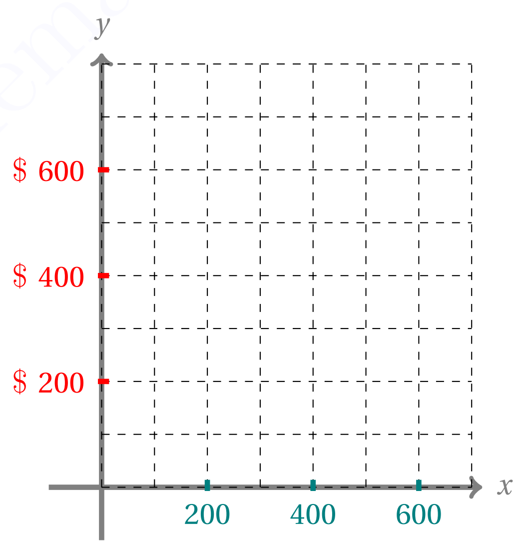
   (b)
   $$\begin{cases} 2x & +3y & = -1 \\ -4x & -6y & = -2 \end{cases}$$

   (c)
   $$\begin{cases} 2x & +3y & = -1 \\ -4x & -5y & = 6 \end{cases}$$

   (d)
   $$\begin{cases} 2x & +3y & = -1 \\ -4x & -6y & = 2 \end{cases}$$

3. Remember $P(x) = R(x) - C(x)$ where P is the profit function, $R(x)$ is the revenue function and $C(x)$ is the cost function when x many units are produced.

   (a) Graph two functions $C(x) = 0.85x + 220$ dollars and $R(x) = 1.25x$.



   (b) Find the breaking even point.

   (c) Explain how the profit changes before and after the breaking even point.

4. The sum of two numbers is 34. One number is 6 less than the other. Find the **larger** number.

(A) 14 (B) 20 (C) 26 (D) 32 (E) None of these

5. Majed has a collection of 20 coins, consisting of only nickels and quarters, which has a value of $\$2.85$. If x is the number of nickels (5 cent pieces) and y is the number of quarters (25 cent pieces), which of the following systems of equations can be used to determine x and y?

(A) $\begin{aligned} x+y&=20 \\ 25x+5y&=285 \end{aligned}$

(C) $\begin{aligned} x+\quad y&=285 \\ 25x+\quad 5y&=20 \end{aligned}$

(B) $\begin{aligned} x+\quad y&=20 \\ 5x+\quad 25y&=285 \end{aligned}$

(D) $\begin{aligned} x+y&=285 \\ 5x+25y&=20 \end{aligned}$

6. A fruit stand sells two varieties of strawberries: standard and deluxe. A box of standard strawberries sells for $\$3$, and a box of deluxe strawberries sells for $\$7$. In one day the stand sells 140 boxes of strawberries for a total of $\$660$. How many boxes of each type were sold?

7. Chloe invested a total of $\$5000$, part at 3% simple interest and part at 4% simple interest. At the end of 1 year, the investments had earned \$176 interest. How much was invested at each rate?

8. The admission fee at an amusement park is $\$1.50$ for children and $\$4.00$ for adults. On a certain day, $2,700$ people entered the park, and the admission fees that were collected totaled $\$5,800$. How many children and how many adults were admitted?

9. Solve the following system of equations, showing all of your work.

$$\begin{cases} x & +2y & -z = 7 \\ 2x & & -z = -6 \\ 3x & +5y & +2z = 16 \end{cases}$$

10. Solve the following system of equations, showing all of your work.

$$\begin{cases} x & +2y & -z & = 2 \\ 2x & +y & +z & = 1 \\ x & -y & +2z & = -1 \end{cases}$$

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11. Solve The following system of equations, showing all your work.

$$\begin{cases} x & +2y & -z & = 2 \\ 2x & +y & +z & = 1 \\ x & -y & +2z & = 1 \end{cases}$$

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12. Solve The following system of equations, showing all your work.

$$\begin{cases} x & +2y & -z & = 2 \\ 2x & +y & +z & = 1 \\ x & -y & +z & = 1 \end{cases}$$

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13. A fruit stand carries 3 different varieties of apples. Honey-crisp apples sell for $\$2.95$/pound, Pink Lady apples sell for $\$2.69$/pound, and Red Delicious apples $\$1.59$/pound. In one day, the fruit stand sells 450 pounds of apples for a total of $\$780$. On the same day the number of pounds of Red Delicious apples sold is three times the number of pounds of Honey-crisp apples sold. Let x be the number of pounds of Honey-crisp apples sold, y be the number of pounds of Pink Lady apples sold and z be the number of pounds of Red Delicious apples sold. A system of equations can be set up using three of the following four equations to determine how many pounds of each type of apple were sold. Choose the equation which does **NOT** need to be used.

(a) $x - 3z = 0$

(b) $x + y + z = 450$

(c) $-3x + z = 0$

(d) $2.95x + 2.69y + 1.59z = 780$

14. A gas station sells three types of gas: Regular for $\$2.95$ a gallon, Performance Plus for $\$3.05$ a gallon, and Premium for $\$3.15$ a gallon. On a particular day $4{,}300$ gallons of gas were sold for a total of $\$12{,}995$. Two times as many gallons of Regular as Premium gas were sold. How many gallons of each type of gas were sold that day?

Here are some more systems for your extra practice. The solutions to these will NOT be posted.

1. $x - 2y - 2z = 2$
   $2x + y = 2$
   $4x - 3y - 6z = 2$

2. $x - 2y + z = 7$
   $2x - 5y + 6z = 15$
   $2x - 3y - 2z = 13$

3. $x - 2y + z = 7$
   $2x - 5y + 6z = 15$
   $2x - 3y - 2z = 12$

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